

# A study of pentaquark $\Theta$ state in the chiral $SU(3)$ quark model <sup>1</sup>

F. Huang, Z.Y. Zhang, Y.W. Yu

Institute of High Energy Physics, Beijing 100039, P.R.China

B.S.Zou

CCAST (World Laboratory), P.O. Box 8730, Beijing 100080;

Institute of High Energy Physics, Beijing 100039, P.R.China

## Abstract

The structure of the pentaquark state  $uudd\bar{s}$  is studied in the chiral  $SU(3)$  quark model as well as in the extended chiral  $SU(3)$  quark model, in which the vector meson exchanges are included. Four configurations of  $J^\pi = \frac{1}{2}^-$  and four of  $J^\pi = \frac{1}{2}^+$  are considered. The results show that the isospin  $T = 0$  state is always the lowest one for both  $J^\pi = \frac{1}{2}^-$  and  $J^\pi = \frac{1}{2}^+$  cases in various models. But the theoretical value of the lowest one is still about  $200 - 300 MeV$  higher than the experimental mass of  $\Theta$ . It seems that a dynamical calculation should be done for the further study.

Key words: Pentaquark state, Quark Model, Chiral Symmetry.

## 1 Introduction

Recently, LEPS Collaboration at SPring 8 [1], DIANA Collaboration at ITEP [2], CLAS Collaboration at Jefferson Lab [3] and SAPHIR Collaboration at ELSA [4] report that they observed a new resonance  $\Theta$ , with strangeness quantum number  $S = +1$ . The mass of this  $\Theta$  particle is around  $M_\Theta = 1540 MeV$  and the upper limit of the width is about  $\Gamma_\Theta < 25 MeV$ . Since it has strangeness quantum number  $S = +1$ , it must

---

<sup>1</sup>Project supported by the National Natural Science Foundation of China

be a 5-quark system. The interesting problem is whether it is a strange meson-baryon molecule like state or a pentaquark state. If it is really a pentaquark state, it will be the first multi-quark state people found. There are already many theoretical works to try to explain its properties with various quark models [5, 6, 7] or other approaches [8]. A re-analysis [9] of older experimental data on the  $K^+$ -nucleon elastic scattering process put a more stringent constraint on the width to be  $\Gamma_\Theta < 1\text{MeV}$ . Since the mass of  $\Theta$ ,  $M_\Theta$ , is larger than the sum of nucleon mass and kaon mass,  $M_N + M_K$ , it is not easy to understand why its width is so narrow, unless it has very special quantum numbers. As to the mass of  $\Theta$ , although it is predicted by the original chiral soliton model [10] quite well, there is no concrete calculation from quark model available yet.

In this work, we calculate the energies of the pentaquark states in chiral quark model. Four configurations of  $J^\pi = \frac{1}{2}^-$  and four of  $J^\pi = \frac{1}{2}^+$  are considered. Some qualitative information is obtained: (1) The isoscalar state,  $T = 0$ , is always the lowest one for both cases of  $J^\pi = \frac{1}{2}^-$  and  $J^\pi = \frac{1}{2}^+$ . (2) The calculated results of the extended chiral  $SU(3)$  quark model are quite similar to those of the chiral  $SU(3)$  quark model, when the parameters are taken as what we used in the  $N - N$  scattering calculation [11, 12]. (3) When the size parameter is adjusted to be  $0.6\text{fm}$ , the energy of the lowest state ( $[4]_{orb}[31]_{ts=01}^{\sigma f} \bar{s}$ ,  $LST = 0\frac{1}{2}0$ ,  $J^\pi = \frac{1}{2}^-$ ), is  $1670\text{MeV}$ , still about  $130\text{MeV}$  higher than the experimental value of the  $\Theta$  mass. A dynamical calculation will be done for getting quantitative information of the  $\Theta$  particle's structure.

## 2 Theoretical framework

For a  $4q - \bar{q}$  color singlet system, the  $4q$  wave function includes three parts: orbital, flavor-spin  $SU(3) \times SU(2)$  and color  $SU(3)$  part. In  $\Theta$  particle case, its strangeness is  $+1$ ,  $4q$  part only includes u and d quarks, and the anti-quark is  $\bar{s}$ . Four configurations for  $J^\pi = \frac{1}{2}^-$  are considered, they are: ( $[4]_{orb}[31]_{ts=01}^{\sigma f} \bar{s}$ ,  $LST = 0\frac{1}{2}0$ ,  $J^\pi = \frac{1}{2}^-$ ), ( $[4]_{orb}[31]_{ts=10}^{\sigma f} \bar{s}$ ,  $LST = 0\frac{1}{2}1$ ,  $J^\pi = \frac{1}{2}^-$ ), ( $[4]_{orb}[31]_{ts=11}^{\sigma f} \bar{s}$ ,  $LST = 0\frac{1}{2}1$ ,  $J^\pi = \frac{1}{2}^-$ ) and ( $[4]_{orb}[31]_{ts=21}^{\sigma f} \bar{s}$ ,  $LST = 0\frac{1}{2}2$ ,  $J^\pi = \frac{1}{2}^-$ ). We also considered 4 configurations for  $J^\pi = \frac{1}{2}^+$ : ( $[31]_{orb}[4]_{ts=00}^{\sigma f} \bar{s}$ ,  $LST = 1\frac{1}{2}0$ ,  $J^\pi = \frac{1}{2}^+$ ), ( $[31]_{orb}[4]_{ts=11}^{\sigma f} \bar{s}$ ,  $LST = 1\frac{1}{2}1$ ,  $J^\pi = \frac{1}{2}^+$ ), ( $[31]_{orb}[4]_{ts=11}^{\sigma f} \bar{s}$ ,  $LST = 1\frac{3}{2}1$ ,  $J^\pi = \frac{1}{2}^+$ ) and ( $[31]_{orb}[4]_{ts=22}^{\sigma f} \bar{s}$ ,  $LST = 1\frac{3}{2}2$ ,  $J^\pi = \frac{1}{2}^+$ ). The color part of them is  $[211]^c$ ,

i.e.  $(\lambda\mu)_c = (10)$ , combining (01) of  $\bar{s}$ , the total quantum number in color space is singlet. For  $J^\pi = \frac{1}{2}^-$  states, color  $[211]^c$  with spin-flavor  $[31]^{\sigma f}$  constructs the total anti-symmetric structure of the  $4q$  part, and for  $J^\pi = \frac{1}{2}^+$  states,  $[31]_{orb}$  replaces  $[31]^{\sigma f}$  to make the anti-symmetrization.

In the chiral  $SU(3)$  quark model the Hamiltonian of the system can be written as

$$H = \sum_i T_i - T_G + \sum_{i < j=1-4} V_{ij} + \sum_{i=1-4} V_{i5}, \quad (1)$$

where  $\sum_i T_i - T_G$  is the kinetic energy of the system,  $V_{ij}, i, j = 1-4$  and  $V_{i5}, i = 1-4$  represent the interactions between quark-quark ( $q-q$ ) and quark-anti-quark ( $q-\bar{q}$ ) respectively.

$$V_{ij} = V_{ij}^{conf} + V_{ij}^{OGE} + V_{ij}^{ch}, \quad (2)$$

$V_{ij}^{conf}$  is the confinement potential taken as the quadratic form,

$$V_{ij}^{conf} = -a_{ij}^c (\lambda_i^c \cdot \lambda_j^c) r_{ij}^2 - a_{ij}^{c0} (\lambda_i^c \cdot \lambda_j^c), \quad (3)$$

and  $V_{ij}^{OGE}$  is the one gluon exchange (OGE) interaction,

$$\begin{aligned} V_{ij}^{OGE} = & \frac{1}{4} g_i g_j (\lambda_i^c \cdot \lambda_j^c) \left\{ \frac{1}{r_{ij}} - \frac{\pi}{2} \delta(\vec{r}_{ij}) \left( \frac{1}{m_{qi}^2} + \frac{1}{m_{qj}^2} \right. \right. \\ & \left. \left. + \frac{4}{3} \frac{1}{m_{qi} m_{qj}} (\vec{\sigma}_i \cdot \vec{\sigma}_j) \right) \right\} + V_{tensor}^{OGE} + V_{\ell\vec{s}}^{OGE} \end{aligned} \quad (4)$$

$V_{ij}^{ch}$  represents the interactions from chiral field couplings. In the chiral  $SU(3)$  quark model  $V_{ij}^{ch}$  includes scalar meson exchange  $V_{ij}^s$ , pseudo-scalar meson exchange  $V_{ij}^{ps}$ , and in the extended chiral  $SU(3)$  quark model, vector meson exchange  $V_{ij}^v$  potentials are also included,

$$V_{ij}^{ch} = \sum_{a=0}^8 V_{sa}(\vec{r}_{ij}) + \sum_{a=0}^8 V_{psa}(\vec{r}_{ij}) + \sum_{a=0}^8 V_{va}(\vec{r}_{ij}). \quad (5)$$

Their expressions can be found in Refs.[11, 12]. The interaction between  $q$  and  $\bar{q}$  includes two parts: direct interaction and annihilation part,

$$V_{i\bar{5}} = V_{q\bar{q}}^{dir} + V_{q\bar{q}}^{ann}, \quad (6)$$

$$V_{q\bar{q}}^{dir} = V_{q\bar{q}}^{conf} + V_{q\bar{q}}^{OGE} + V_{q\bar{q}}^{ch}, \quad (7)$$

with

$$V_{q\bar{q}}^{ch}(\vec{r}) = \sum_i (-1)^{G_i} V_{q\bar{q}}^{ch,i}(\vec{r}). \quad (8)$$

Here  $(-1)^{G_i}$  describes the G parity of the  $i$ th meson. For the  $\Theta$  particle case,  $q\bar{q}$  can only annihilate into  $K$  and  $K^*$  mesons, thus  $V_{i5}^{ann}$  can be expressed as:

$$V_{i5}^{ann} = V_{ann}^K + V_{ann}^{K^*}, \quad (9)$$

with

$$V_{ann}^K = \tilde{g}_{ch}^2 \frac{1}{(\tilde{m} + \tilde{m}_s)^2 - m_K^2} \left( \frac{1 - \vec{\sigma}_q \cdot \vec{\sigma}_{\bar{q}}}{2} \right)_{spin} \left( \frac{2 + 3\lambda_q \cdot \lambda_{\bar{q}}^*}{6} \right)_{color} \left( \frac{19}{9} + \frac{1}{6} \lambda_q \cdot \lambda_{\bar{q}}^* \right)_{flavor} \delta(\vec{r}_q - \vec{r}_{\bar{q}}), \quad (10)$$

and

$$V_{ann}^{K^*} = \tilde{g}_{chv}^2 \frac{1}{(\tilde{m} + \tilde{m}_s)^2 - m_{K^*}^2} \left( \frac{3 + \vec{\sigma}_q \cdot \vec{\sigma}_{\bar{q}}}{2} \right)_{spin} \left( \frac{2 + 3\lambda_q \cdot \lambda_{\bar{q}}^*}{6} \right)_{color} \left( \frac{19}{9} + \frac{1}{6} \lambda_q \cdot \lambda_{\bar{q}}^* \right)_{flavor} \delta(\vec{r}_q - \vec{r}_{\bar{q}}). \quad (11)$$

Where  $\tilde{g}_{ch}$  and  $\tilde{g}_{chv}$  are the coupling constants of pseudo-scalar-scalar chiral field and vector chiral field in the annihilation case respectively.  $\tilde{m}$  represents the effective quark mass. Actually,  $\tilde{m}$  is quark momentum dependent, here we treat it as an effective mass.

Using these two models, we did an adiabatic approximation calculation to study the energies of the  $(uudd-\bar{s})$  system.

### 3 Results and discussions

First, we carry on the calculation by taking the parameters which can reasonably reproduce the experimental data of  $N - N$  and  $Y - N$  scattering [11, 12]. In the chiral  $SU(3)$  quark model, besides pseudo-scalar and scalar fields coupling,  $OGE$  interaction is still there to offer part of the short range repulsion, as well as in the extended chiral  $SU(3)$  quark model, the  $OGE$  interaction is almost replaced by the vector meson exchanges. About the annihilation interaction between  $u(d) - \bar{s}$ , it is a complicated problem, in Eqs. (10) and (11), the quark effective masses  $\tilde{m}$  and  $\tilde{m}_s$ , as well as the annihilation coupling constants  $\tilde{g}_{ch}$  and  $\tilde{g}_{chv}$  are subject to significant uncertainties. In

our calculation, we treat  $(\tilde{m} + \tilde{m}_s)$ ,  $\tilde{g}_{ch}$  and  $\tilde{g}_{chv}$  as parameters, and adjust them to fit the masses of  $K$  and  $K^*$  mesons, named case I. In case II, we omitted the annihilation interaction in the calculation to see its effects. All results of 4 configurations of  $J^\pi = \frac{1}{2}^-$  and 4 of  $J^\pi = \frac{1}{2}^+$  are listed in Table 1.

From Table 1, one can see that: (1) The isoscalar state ( $T = 0$ ) is always the lowest state both in  $J^\pi = \frac{1}{2}^-$  and in  $J^\pi = \frac{1}{2}^+$  cases, and  $([4]_{orb} [31]_{ts=01}^{\sigma f} \bar{s}, LST = 0\frac{1}{2}0, J^\pi = \frac{1}{2}^-)$  is always the lowest one in different models. (2) The results of the chiral  $SU(3)$  quark model and the extended chiral  $SU(3)$  quark model are quite similar, although the short range interactions of these two models are different, one is from  $OGE$  and the other is from vector meson exchanges. (3) The annihilation interactions offer attraction to the states of  $J^\pi = \frac{1}{2}^-$  and repulsion to  $J^\pi = \frac{1}{2}^+$  states. (4) When the annihilation interaction is considered, the energy of the lowest state,  $([4]_{orb} [31]_{ts=01}^{\sigma f} \bar{s}, LST = 0\frac{1}{2}0, J^\pi = \frac{1}{2}^-)$ , is about  $250 - 300 MeV$  higher than the experimental value of the  $\Theta$  mass.

We tried to adjust the size parameter  $b_u$  to be larger to see the influence. As an example, the results of  $b_u = 0.6 fm$  in the chiral  $SU(3)$  quark model are given in Table 2. In this case, the energies of all states become smaller, caused by the kinetic energy of the system is reduced for larger  $b_u$ . When the annihilation interaction is included (case I), the energy of the lowest state,  $([4]_{orb} [31]_{ts=01}^{\sigma f} \bar{s}, LST = 0\frac{1}{2}0, J^\pi = \frac{1}{2}^-)$ , is  $1670 MeV$ , about  $130 MeV$  higher than the  $\Theta'$  mass.

In our results, the states of  $J^\pi = \frac{1}{2}^-$  are always lower than those of  $J^\pi = \frac{1}{2}^+$ , even in the extended chiral  $SU(3)$  quark model, in which the  $OGE$  interaction is almost totally replaced by vector meson exchanges. According to Stancu and Riska's argument [6], the state of  $T = 0, J^\pi = \frac{1}{2}^+$  can be lower than the state of  $T = 0, J^\pi = \frac{1}{2}^-$ , because the spin-flavor dependent interactions from Goldstone-Boson exchange potential offer more attractions to the state of  $T = 0, J^\pi = \frac{1}{2}^+$ . In our calculation, it is true that  $\pi$  and  $\rho$  meson exchanges do contribute very strong attractions to the state of  $T = 0, J^\pi = \frac{1}{2}^+$ , but when the interactions between  $u(d)$  and  $\bar{s}$  are included, especially the annihilation terms are considered, the state of  $T = 0, J^\pi = \frac{1}{2}^-$  gets more attractions. This is because that among 4 pairs  $u(d) - \bar{s}$  interactions, the state of  $T = 0, J^\pi = \frac{1}{2}^-$  has 1

Table 1: Energies of pentaquark states in different chiral quark model

configuration	Chiral $SU(3)$ Quark Model $b_u=0.50$ fm		Ex. Chiral $SU(3)$ Quark Model $b_u=0.45$ fm	
$J^\pi = \frac{1}{2}^-$	I	II	I	II
	(MeV)		(MeV)	
$[4]_{orb}[31]_{ts=01}^{\sigma f} \bar{s}$	1801	1957	1843	2091
$[4]_{orb}[31]_{ts=10}^{\sigma f} \bar{s}$	2049	2128	2089	2170
$[4]_{orb}[31]_{ts=11}^{\sigma f} \bar{s}$	2117	2190	2115	2193
$[4]_{orb}[31]_{ts=21}^{\sigma f} \bar{s}$	2323	2369	2314	2334
$J^\pi = \frac{1}{2}^+$	I	II	I	II
	(MeV)		(MeV)	
$[31]_{orb}[4]_{ts=00}^{\sigma f} \bar{s}$	2271	2185	2270	2253
$[31]_{orb}[4]_{ts=11}^{\sigma f} \bar{s}$	2308	2235	2296	2310
$(S = \frac{1}{2})$				
$[31]_{orb}[4]_{ts=11}^{\sigma f} \bar{s}$	2362	2282	2367	2337
$(S = \frac{3}{2})$				
$[31]_{orb}[4]_{ts=22}^{\sigma f} \bar{s}$	2426	2367	2412	2435

Table 2: Energies of pentaquark states in chiral  $SU(3)$  quark model with  $b_u = 0.6fm$ 

configuration	Chiral $SU(3)$ Quark Model ( $b_u = 0.60fm$ )		
$J^\pi = \frac{1}{2}^-$	I	II	III
	(MeV)		
$[4]_{orb}[31]_{ts=01}^{\sigma f} \bar{s}$	1672	1867	2027
$[4]_{orb}[31]_{ts=10}^{\sigma f} \bar{s}$	1940	1990	2039
$[4]_{orb}[31]_{ts=11}^{\sigma f} \bar{s}$	1983	2026	2051
$[4]_{orb}[31]_{ts=21}^{\sigma f} \bar{s}$	2103	2124	2090
$J^\pi = \frac{1}{2}^+$	I	II	III
	(MeV)		
$[31]_{orb}[4]_{ts=00}^{\sigma f} \bar{s}$	2105	2016	1997
$[31]_{orb}[4]_{ts=11}^{\sigma f} \bar{s}$ ( $S = \frac{1}{2}$ )	2124	2051	2018
$[31]_{orb}[4]_{ts=11}^{\sigma f} \bar{s}$ ( $S = \frac{3}{2}$ )	2145	2070	2018
$[31]_{orb}[4]_{ts=22}^{\sigma f} \bar{s}$	2177	2122	2051

pair  $u - \bar{s}$  of  $(0s)^2$  with spin  $s = 0$  and color singlet  $(00)_c$  (i.e.  $K$  meson's quantum numbers) and  $\frac{1}{3}$  pair of  $(0s)^2$   $s = 1$   $(00)_c$ , the other part is color octet, but the state of  $T = 0, J^\pi = \frac{1}{2}^+$  only has  $\frac{1}{12}$  pair of  $(0s)^2$   $s = 0$   $(00)_c$ ,  $\frac{1}{4}$  pair of  $(0s0p)$   $s = 0$   $(00)_c$ ,  $\frac{1}{4}$  pair of  $(0s)^2$   $s = 1$   $(00)_c$ ,  $\frac{3}{4}$  pair of  $(0s0p)$   $s = 1$   $(00)_c$  and the other part is color octet. If we take the annihilation interaction to fit the masses of  $K$  and  $K^*$ , the state of  $T = 0, J^\pi = \frac{1}{2}^-$  must be the lowest. In Table 2, case II is of the annihilation interactions omitted, and the results of without any interactions between  $4q$  and  $\bar{s}$  are given in column III. One sees from Table 2 that when the interactions between  $4q$  and  $\bar{s}$  are all omitted, the state of  $T = 0, J^\pi = \frac{1}{2}^+$  can be the lowest one, but its energy ( $1997MeV$ ) is much higher than the  $\Theta$ 's mass.

## 4 Conclusions.

The structures of pentaquark states are studied by an adiabatic approximation calculation in the chiral quark model. When the interactions between  $4q$  and  $\bar{s}$  are considered, especially the parameters in the annihilation interactions are fixed by fitting the masses of  $K$  and  $K^*$ , our results show that state  $T = 0, J^\pi = \frac{1}{2}^-$  is the lowest one, and its energy is about  $150-300\text{MeV}$  higher than the  $\Theta$ 's mass. If we omit the interactions between  $4q$  and  $\bar{s}$ , then the state  $T = 0, J^\pi = \frac{1}{2}^+$  can be lower than state  $T = 0, J^\pi = \frac{1}{2}^-$ , in agreement with what is claimed in Ref.[6]. But the mass will be more than 400 MeV higher than the observed  $\Theta$  mass. This means that how to treat the annihilation interaction reasonably is very important in the calculation. On the other hand, even without omitting the interactions between  $4q$  and  $\bar{s}$ , all of the results of various models with different parameters in this adiabatic approximation calculation still give the lowest mass for the  $uudd\bar{s}$  pentaquark at least more than 150 MeV above the observed mass of  $\Theta$ . Furthermore, its  $T = 0, J^\pi = \frac{1}{2}^-$  configuration has a potential s-wave KN fall-apart mode [7] and hence is very difficult to explain the very narrow width of the observed width of  $\Theta$ . It seems that it is impossible to reproduce the observed low mass and narrow width of  $\Theta$  by quark models with reasonable model parameters in the adiabatic approximation and a dynamical calculation may be necessary for the further study.



## References

- [1] T. Nakano, D.S. Ahn, et. al., *Phys. Rev. Lett.*, **91**, 2003, 012002.
- [2] V.V. Barmin et. al., *hep-ex/0304040*.
- [3] S. Stepanyan, K. Hicks, et. al., *hep-ex/0307018 v3*, 16 Jul 2003.
- [4] J. Barth, W. Braun, et. al., *hep-ex/0307083 v3*, 6 Aug 2003.
- [5] Simon Capstick and Philip R. Page, *hep-ph/0307019 v2*, 7 Aug 2003.
- [6] FI. Stancu and D.O. Riska, *hep-ph/0307010 v1*, 1 Jul 2003;  
L.Ya Glozman, *hep-ph/0308232*.
- [7] B.K. Jennings and K. Maltman, *hep-ph/0308286*.
- [8] R. Jaffe and F. Wilczek, *hep-ph/0307341*;  
F.Csikor et al, *hep-lat/0309090*;  
S.L. Zhu, *hep-ph/0307345*.
- [9] R.A. Arndt et. al., *nucl-th/0308012*
- [10] D. Diakonov, V. Petnov and M. Polyakov, *Z. Phys.*, A359, 1997, 305.
- [11] Z.Y. Zhang, Y.W. Yu, P.N. Shen, L.R. Dai, A.Faessler and U.Straub,  
*Nucl. Phys.*, **A625**, 1997, 59.
- [12] L.R. Dai, Z.Y. Zhang, Y.W. Yu and P. Wang, *Nucl. Phys.*, **A727**, 2003, 321.